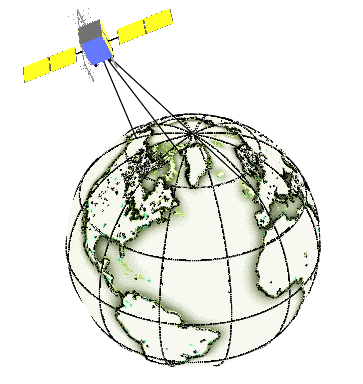
**Astronautical Engineering 422**

**Project Cowell**

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- **Introduction**

The cowell.m function is designed to numerically propagate an orbit with perturbations using a fourth order Runga-Kutta integrator. The perturbations modeled were atmospheric drag, oblate earth effects, 3rd body effects, and solar radiation pressure.

o Objectives

o Approach

- **Assumptions, Math Techniques and Results**

Several assumptions were used in perturbation modeling for cowell.m:

Drag

For drag, density was derived from a 1976 standard atmosphere model. The relative velocity of the atmosphere to the spacecraft was calculated under the assumption that the atmosphere moved at the same rate as the surface beneath it. Finally, the spacecraft’s cross sectional area and coefficient of drag were assumed to be constant, implying one surface constantly tracking the direction of movement. Because drag is the most significant of the perturbing forces—as seen later in this report—these assumptions are the most critical. The assumption about the velocity of the atmosphere can be considered minor, as the magnitude of the spacecraft’s velocity is far more substantial. The assumptions made with respect to atmospheric density, cross sectional area of the spacecraft, and coefficient of drag, however are significant, and were made out of a necessity for simplification.

Geopotential

Geopotential effects of the earth were modeled with World Geodetic System 1984 (WGS84) constants for the zonal terms, J2, J3, and J4.

Third Body Effects

The sun and moon were both modeled with the cowell.m function. The relative positions of both of these bodies with respect to the earth were modeled with sub-functions, sun.m and moon.m, each with an assumed level of accuracy. The gravitational constants of the bodies were taken from WGS84 data. Each body was assumed to be uniformly dense and spherical, so geopotential effects of the third bodies need not be considered. As the distance to the sun or moon from the spacecraft will likely be immense for all reasonable applications, this is a reasonable assumption.

Solar Radiation Pressure

Solar radiation pressure modeling carries assumptions similar to those made with other perturbations. Cross sectional area and coefficient of reflectivity were assumed constant, and behaved much the same as drag. While drag force was applied in the negative direction of spacecraft motion, however, solar radiation was applied in the negative direction of the satellite to sun vector, requiring the use of the sun.m function again. Solar radiation pressure was unique in its need for a conditional application of the force. While the earth is between the sun and the spacecraft, the spacecraft encounters no solar radiation pressure, so a cylindrical shadow was modeled behind the earth. This assumption become less valid as the orbit becomes larger because the sun is a finite distance from the earth and in reality forms a conic shadow.

Results

The following table summarizes the results a test orbit with several sets of perturbations. The orbit had the following orbital elements and characteristics at 1200 UT on 1 Jan 2015.

a = 6633.262Km

e = 0.02

i = 45.0

Ω = 45.0

ω = 90.0

υ = 0.0

Satellite mass = 12,000 kg

Satellite frontal area and reflectivity area = 16.4 m2

Satellite drag coefficient, CD = 2.0

Satellite coefficient of reflectivity, CR = 1.0

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Summary of Perturbated COEs** | | | | | | | | |
| **Case** | **Perturbations** | **Semimajor Axis (km)** | **Eccentricity** | **Inclination (deg)** | **Right ascension of the ascending node (deg)** | **Argument of Perigee (deg)** | **True Anomaly (deg)** | **Time (s)** |
| a | *None* | *6633.2620* | *0.020000* | *45.00000* | *45.0000* | *90.0000* | *0.0000* | *5376.5* |
| b | *Drag* | *6632.5609* | *0.019906* | *44.99998* | *45.0000* | *90.0003* | *0.0293* | *5376.5* |
| c | *J2* | *6633.2620* | *0.020000* | *45.00000* | *44.6181* | *90.4063* | *359.6996* | *5376.5* |
| d | *J3* | *6633.2620* | *0.020000* | *45.00000* | *45.0000* | *89.9838* | *0.0164* | *5376.5* |
| e | *J4* | *6633.2620* | *0.020000* | *45.00000* | *45.0002* | *90.0132* | *359.9952* | *5376.5* |
| f | *Drag, J2* | *6632.5796* | *0.019908* | *44.99998* | *44.6180* | *90.4054* | *359.7296* | *5376.5* |
| g | *Sun* | *6633.2620* | *0.020000* | *45.00000* | *45.0000* | *90.0000* | *360.0000* | *5376.5* |
| h | *Moon* | *6633.2620* | *0.020000* | *45.00000* | *45.0000* | *89.9999* | *0.0000* | *5376.5* |
| i | *SRP* | *6633.2620* | *0.019998* | *45.00001* | *45.0000* | *89.9923* | *0.0084* | *5376.5* |
| k | *Drag, J2, J3, J4, Sun, Moon* | *6632.5789* | *0.019908* | *44.99998* | *44.6181* | *90.4022* | *359.7412* | *5376.5* |

To validate the accuracy of the function, it was necessary to compare the results to an established tool. Case f, drag and J2 was used for this purpose because these two perturbations were the most significant, and Satellite Tools Kit was the established tool was used as the benchmark.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Case f: Drag, J2** | | | |  | **Case f: Drag, J2** | | | |
| **COEs** | **Matlab** | **STK** | **Difference** |  | **R** | **Matlab** | **STK** | **Difference** |
| Semimajor Axis (km) | *6632.579556* | 6632.577608 | 0.001948 |  | I (km) | *-3240.108413* | -3239.4225 | 0.685881 |
| Eccentricity | *0.019908* | 0.019908 | 0.000000 |  | J (km) | *3260.418672* | 3261.0979 | 0.679257 |
| Inclination (deg) | *44.99998* | 45 | 0.000020 |  | K (km) | *4596.557136* | 4596.5586 | 0.001502 |
| Right ascension of the ascending node (deg) | *44.617953* | 44.618 | 0.000047 |  | **V** |  |  |  |
| Argument of Perigee (deg) | *90.405409* | 90.405 | 0.000409 |  | I (km/s) | *-5.618854* | -5.61943 | 0.000576 |
| True Anomaly (deg) | *359.72959* | 359.73 | 0.000410 |  | J (km/s) | *-5.564809* | -5.564227 | 0.000582 |
| Time (s) | *5376.5* | 5376.5 | 0.0 |  | K (km/s) | *-0.014494* | -0.013696 | 0.000798 |

- **Discussion/Conclusions/Recommendations**

Most of the variance between cowell.m and the STK can be attributed to round off error or a difference in precision being carried through calculations. The most significant of these differences, as well as the most concerning, is the positional information of the satellite, or R vector. The error has been verified to not be caused by a difference in time of propagation or the Runga-Kutta integrator through close inspection of step counts and two body propagation respectively. Further testing is needed to determine if it constitutes a flaw in the cowell.m function.

Step size is always a concern with a numerical propagator. For the purposes of this report, a 5 second step was used, but other options were investigated. Examining only the I component of the spacecraft’s positional vector, an association between of step size, accuracy, and computation time was developed. The following table and graph outline the results.

|  |  |  |
| --- | --- | --- |
| Time Step Characterization | | |
| Step Size (s) | Magnitude of R I (km) | Time |
| 100 | 3241.096 | 0.331 |
| 50 | 3240.159 | 0.627 |
| 25 | 3240.111 | 1.446 |
| 10 | 3240.108 | 2.930 |
| 5 | 3240.108 | 4.633 |
| 2.5 | 3240.108 | 9.141 |
| 1 | 3240.108 | 21.762 |
| 0.5 | 3240.108 | 42.825 |
| 0.1 | 3240.108 | 170.312 |

The primary concerns with step size are accuracy and computing time. Smaller steps generally lead to more accurate results, but at very small step sizes, round off error will become more apparent. Double-precision float values lose miniscule pieces of data in each calculation, and while smaller steps yield diminishing returns, this round off error is linearly proportional to the number of times a computation is carried out. At smaller step sizes, cowell.m appears to converge on a value near 3240.108 kilometers. The most apparent conclusion is that a step size of 10 seconds—twice the previously used value of 5 seconds—yields a value within one meter of a step size of 0.1 seconds in only 2.93 seconds as opposed to 170.312. For single orbits the time difference might be trivial, but for long propagations a longer step size is advisable.

**References**

[1] "Fundamentals of Astrodynamics and Applications, 4rd Edition", Vallado

[2] "Fundamentals of Astronautics", Bate, Mueller and White

Documentation: None

**Appendices**

CowellTest.txt: Example of cowelltest.m output, Case F (drag and J2).

cowell.m: cowell.m code.

cowelltest.m: cowelltest.m code.

pderiv.m: pderiv.m code.